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Liquidity and Portfolio Optimization

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Abstract

Liquidity, within the context of defining an optimal portfolio of risky assets, may be viewed as a non-linear return penalty factor that depends on the level of investment and asset size or float.

Liquidity issues often have important implications for theoretical and practical finance. For example, models of capital market equilibrium typically omit the role liquidity plays in the functioning of capital markets in an institutional context (Allen 2002). Liquidity issues may often rationalize the occurrence of panics and bubbles (Allen and Gale 2003). Security valuation and risk estimation is typically associated with liquidity factors. Liquidity is also an important component for devising effective trading strategies.

It is beyond the scope of this short note to consider the many open issues involving liquidity in modern finance. We limit our focus to liquidity within the context of defining an optimal portfolio of risky assets.

The impact of illiquidity in a portfolio optimization may often be viewed as a non-linear return penalty factor that depends on the level of investment and asset size or float. New Frontier Advisors now includes a liquidity or quadratic transaction cost option in our asset allocation software that we recommend using whenever asset universes have heterogeneous liquidity.[1] We assume a Michaud resampled mean-variance optimization and efficient frontier in our discussion.[2] Our objective is to provide guidance on understanding liquidity issues and on using the new liquidity options available in our asset allocation software.

Portfolio Optimization and Illiquid Assets

Liquidity is often a first order factor for defining an investment meaningful portfolio optimization. Many portfolio optimization puzzles can be traced to omission of the liquidity factor. Illiquid assets often have putatively high estimated returns and low measured risk. This is in part because the risk of illiquid assets is often hard to measure without advanced statistical methods. An allocation study that includes the S&P500 index with private equity or hedge funds will often lead to investment irrelevant portfolios without an appropriate adjustment for relative liquidity. Cases of illiquid asset interest also include small capitalization stocks, real estate, and emerging markets. Even for developed capital markets, including small markets such as Austria with the U.S. or U.K. equity markets may often result in investment irrelevant optimal portfolios. When considering illiquid assets in an asset allocation, it is more than normally important that the asset universe is thoughtfully considered and includes a suitable mixture of liquid assets as well.

Liquidity Dimensions

The illiquidity of an asset can often be conceptualized in a portfolio optimization as an increasing non-linear return penalty function of the allocation. As allocation to the asset increases, the return penalty increases non-linearly. In practice, it is convenient to model non-linear increasing illiquidity penalties with a quadratic function in a mean-variance optimization. Liquidity in a portfolio optimization has a number of dimensions including

[1] Newsletter, February 2003.
[2] Michaud (1998, Ch. 6).

asset characteristics, portfolio or investment size dependency, subjective assessments, and investable universe.

Asset Characteristics

An important asset liquidity characteristic is time to trade. Real estate, for example, is typically considered illiquid relative to exchange traded equity securities because the time to trade can be long. Private equity and some hedge fund strategies also have systemically long-term time return horizons and lockups.

Another liquidity characteristic is capital size of the asset. Small capitalization stocks, emerging markets, and some hedge fund strategies have capital limitations that may non-linearly affect estimated return as allocations increase.

A third liquidity asset characteristic is transparency. Some hedge funds, investment strategies, and private equity investments may be considered illiquid because of the lack of financial transparency and the difficulty of estimating risk reliably. Increasing the allocation to such assets may imply a non-linear increase in perceived risk or disutility that may be approximated as a non-linear reduction in estimated return.

Portfolio Dependent Characteristics

Liquidity is portfolio context dependent. A 10% asset allocation with a $100,000 capital investment may represent very different liquidity considerations relative to a $10 billion dollar cash investment. For a starting portfolio of risky assets, the return penalty is measured relative to current portfolio weights. When there is no risky asset portfolio, a cash asset with zero risk and return is assumed that fills the role of a current portfolio in assessing liquidity. The total capital value of either investment is a critical factor.

Combinations of cash and risky asset portfolios may also be considered as the basis for defining the liquidity penalty. For an investor with some proportion in a risky asset portfolio and the remainder in cash, the risky asset portfolio weights should be input so that the sum reflects the proportion of capital value of the risky portfolio and the optimization then includes liquidity penalties relative to the total investment of risky portfolio and cash.

Subjective Return Penalties

Illiquidity penalties are usually conceptualized as non-linear transaction costs. However, illiquidity penalties may also represent subjective estimates or “utiles” of return penalties or disutilities. Such a concept may be useful for modeling non-linear penalties for non-transparent assets.

An Expanded Universe

An alternative approach to illiquidity that does not use the notion of penalty functions is that of an expanded (or contracted) investment universe. Suppose a private equity asset of a firm whose characteristics are similar to that of an exchange-traded stock. In this case estimated return and risk may be approximately the same. Now consider the utility function of a long-term investor such as endowment funds and foundations. Long-term

investors may be indifferent to the illiquidity associated with long-term return lockups while more typical investors may find such assets less desirable. The effect of return lockup indifference for some classes of investors is to expand the universe of investable assets resulting in an enhanced risk-reward tradeoff. Differences in utility perceptions often provide a useful framework for rationalizing differences in investable universes of various market participants.

Graphing the Liquidity Constrained Efficient Frontier

When the quadratic transactions cost option is used, the graphical display of the resampled efficient frontier is adjusted for return liquidity constraints.[3] Conceptually, the difference in the constrained case is that estimated return is reduced for illiquid assets and the efficient frontier shifts downwards in parts of the mean-variance framework. This is the same approach traditionally used in the case of more common linear trading costs. Portfolio risk is unaffected. The properties of resampled efficient frontiers do not change.

The benefits of the liquidity constrained optimization are often best seen in the resampled portfolio composition maps. An option will be available to graph the constrained efficient asset allocations without the liquidity constraints.[4] Note, however, that in the unconstrained case, the familiar monotone increasing risk-return relationship associated with mean-variance efficiency may not exist.

Using the NFA Quadratic Penalty Option

Release 3.3.1 of NFA's asset allocation software, introduced in February 2003, includes a quadratic penalty transaction cost option appropriate for asset allocation studies. This version of the software also includes an option to customize the power of the quadratic penalty function.

The quadratic transaction cost option is available on the input sheet of the optimizer software. Users may input quadratic return penalties for each individual asset.

Liquidity quadratic penalties require calibration relative to the notion of a "large" asset weight that serves as a reference point. Such considerations may vary by information set and investable universe. The quadratic reference point, an input on the input sheet of the NFA optimizer program, is the asset allocation that equates linear and quadratic/liquidity costs. The quadratic reference point defines the intersection of linear trading and quadratic cost functions, for equal return penalties. At larger asset allocations relative to the reference point, the quadratic/liquidity option dominates; at smaller asset weights relative to the reference point a linear trading cost penalty dominates.

Summary

Liquidity is an important consideration for many applications of portfolio optimization and optimal asset allocation. Few portfolio optimizations are based on a uniformly liquid asset universe. Liquidity has been the missing factor in many studies and a primary cause

[3] This is the default display in all future NFA software releases.
[4] This option will be on the input page of the optimizer module.

of investment unintuitive and irrelevant optimized portfolios. Appropriately defined quadratic return penalties may make the difference between an asset allocation that is of great investment value and one that is irrelevant. Users of NFA's optimization software have the tools for effectively including a wide variety of liquidity considerations in a portfolio optimization study. With experience, the new options may often provide real investment value to the asset management process.

References

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