Comment on: Kritzman, M. 2006, “Are Optimizers Error Maximizers?”

By

Richard O. Michaud[1]

Draft: November 2019

Forthcoming in the Journal of Portfolio Management

© 2019 New Frontier Advisors, LLC. All rights reserved.

Please do not copy or reproduce in any form without permission of the authors.

[1] Richard O. Michaud is CEO of New Frontier Advisors, LLC, Boston, MA 02110.

Kritzman (2006) asserts that the widely cited Michaud (1989) study characterizing the impact of estimation error on Markowitz (1952) mean-variance (MV) optimization as "error maximization" is hype. His paper consists of two examples of MV portfolio optimizations: an eight-country index asset allocation with near identical returns and four-indices with very different estimates. In spite of the absence of standard references in estimation error, authors continue to cite the paper.[2] In this brief note we demonstrate that even in these stylized and unrealistic examples, Kritzman's MV optimized portfolios perform on average worse than equal weighting out-of-sample. A MV optimization worse than equal weighting has little practical investment value or interest. The impact of optimizer error maximization properly measured appears alive and very well.

We use the iconic Jobson and Korkie (JK) (1981) and Frost and Savarino (FS) (1988) simulation study frameworks to properly measure the impact of estimation error on average out-of-sample value for the Kritzman cases.[3,4] The first row in the Table below displays the in-sample (referee) value of quadratic utility for the three risk aversion (lambda) parameters 0.5, 1.0, and 2.0 in the Kritzman study; the second for the equal-weighted portfolio utility; the third for the simulated average out-of-sample referee-scored utility. For each risk aversion parameter equal-weighted portfolio utility is greater than average MV optimized utility. For the more traditional average Sharpe ratio criteria, the in-sample Sharpe ratio is 37%, average out-of-sample 30%, and equal weighted portfolio 35%. Equal weighting is far superior to MV optimization in all these cases. In the 4-asset case the average out-of-sample utility is diminished substantially and little different from the equal-weighted portfolio.

Effective investment technology should enable outperformance relative to less informed peers and beyond equal weighting. When measured correctly, even atypical MV optimized asset allocations often suffer serious competitive limitations from the investment consequences of error maximization. Error maximization is no hype but an unforgiving reality for practitioners affecting the investment value of MV optimized portfolios in applications without refinements.[5]

Optimized Utilities: Eight Country Case (Annualized)

	Lambda = 0.5	Lambda = 1	Lambda = 2
In-sample Utility	4.6	3.3	0.9
Equal Weighted Utility	4.5	3.0	0.1
Average Out-of-sample Utility	3.4	1.3	-2.0

Richard O. Michaud, PhD is CEO of New Frontier Advisors, Boston, MA 02110
October 2019

[2] For example, Menchero and Ji (2019) and Allen et al (2019).

[3] Inputs are Kritzman returns and covariance matrix computed from monthly returns for the historical period of the examples. We verified that our results are consistent with Kritzman's. Following JK, the simulations assumed 60-monthly returns, with zero risk free rate in the Sharpe ratio computation.

[4] The textbook Judge et al (1988) specifically mentions Monte Carlo simulation as a commonly used general method for dealing with inequality restricted statistical estimation.

[5] An example is Michaud optimization defined in Michaud and Michaud (2008 Ch. 6).

References

Allen, D., C. Lizieri, S. Satchel 2019. "In Defense of Portfolio Optimization: What If We Can Forecast?" *Financial Analysts Journal*, 75(3):20-38.

Frost, P., and Savarino, J., 1986. "An Empirical Bayes Approach to Efficient Portfolio Selection." *Journal of Financial and Quantitative Analysis, 21*(3): 293-305.

Jobson, J., and Korkie, R., 1981. "Putting Markowitz theory to work." *The Journal of Portfolio Management,* 7(4): 70-74.

Judge, G., R.C. Hill, W. Griffiths, H. Lutkepohl, T-C Lee, 1988. *Introduction to the Theory and Practice of Econometrics*, 2nd ed. Wiley, New York.

Markowitz, H. (1952). "Portfolio selection." *The Journal of Finance*, 7(1): 77-91.

Menchero, J, and Ji, L., 2019. "Portfolio Optimization with Noisy Covariance Matrices." *Journal of Investment Management*, 17(1): 77-91.

Michaud, R., 1989. "The Markowitz Optimization Enigma: Is Optimization Optimal?" *Financial Analysts Journal,* 45(1), 31–42.

Michaud, R. and R. Michaud, 2008. *Efficient Asset Management: A Practical Guide to Stock Portfolio Optimization and Asset Allocation*. Oxford University Press, New York. 1st ed. 1998, originally published by Harvard Business School Press, Boston.