Comment: Allen, D., C. Lizieri, S. Satchell 2019. "In Defense of Portfolio Optimization: What If We Can Forecast?"

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Allen et al (ALS) (2019) claim that a CAPM based theoretical framework for Markowitz (1952) mean-variance (MV) efficiency and a small level of forecast information (IC) can beat equal weighted portfolios. A portfolio optimization procedure worse than equal weighting would have little practical investment value or interest. They challenge the 1/N empirical results in DeMiguel et al (DGU) (2009) and, implicitly, the "error maximization" characterization of MV optimization in Michaud (1989). However, their conclusions are inconsistent with canonical Monte Carlo simulation studies of estimation error in MV optimization. This is because their theoretical CAPM-like framework ignores the bulk of estimation error – model error and covariance matrix estimation – by assumption. Our extension of classic Monte Carlo studies indicates that many times the level of forecast information assumed in ALS is likely required to outperform equal weight in theoretical budget-constrained MV optimized portfolios in practice.

Some background may be useful. The phrase "error maximization" characterizes the nearly ubiquitous investment practice of adding ad hoc constraints and refinements of risk-return inputs to engineer investment and marketing "acceptable" MV optimized portfolios. Small changes in inputs may often lead to large changes in the MV optimized portfolio.[2] Even when sophisticated methods are used to refine the statistical character of estimates, the instability of the procedure often leads to ambiguity for defining the MV "optimal" portfolio.[3] Note that the authors find it necessary to inequality constrain the optimizer in their back tests. A brief review of classic Monte Carlo simulation studies of estimation error in MV portfolio optimization provides useful guidance.

Jobson and Korkie (JK) (1981) provide the first rigorous application of Monte Carlo simulation methods for measuring the impact of estimation error in MV portfolio optimization.[4] The JK 20-stock universe study demonstrated that equal weighting dramatically beats budget-constrained MV optimization on average out-of-sample. JK provides a highly relevant rationale for the 1/N results in DGU.

The Frost and Savarino (FS) (1988) Monte Carlo simulation study for a 200-stock universe finds that equal weighting beats budget-constrained MV optimization. FS notes that sign-constrained and inequality constraints, virtually universal in applications, may improve out-of-sample performance of MV optimized portfolios relative to equal weighting.

Michaud et al (MEM) (2019) provides a generalization of the JK and FS simulation studies for five to five hundred stock optimization universes in the case of equal weight, budget-constrained, and sign-constrained MV optimization for different levels of forecast information. MEM shows that, for an IC of 0.1, equal weighting beats budget-constrained MV optimization. The simulations are engineered to have a constant IC even as the optimization universe consists of increasing numbers of stocks. The MEM results indicate that an IC level of 0.3 or higher may lead to budget-constrained MV optimization outperforming equal weight portfolios. Unfortunately, a level of IC less than 0.1 is required in the Grinold (1989) theoretical model used in ALS.

In general, the impact of estimation error in MV optimization increases as the size of the universe increases. Consequently, it should not be surprising that the JK results on the limitations of budget-constrained MV optimization as a practical tool of asset management are consistent with FS and MEM.

[2] Even in cases of "close substitutes," Monte Carlo simulation of the impact of estimation error in MV portfolio optimization may result in inferior average out-of-sample performance relative to equal-weighted portfolios (Michaud 2019).

[3] For example Ledoit and Wolfe (2004).

[4] JK describe the basic Monte Carlo simulation framework.

The ALS theoretical framework implies a level of information far greater than is theoretically valid for their assumptions and likely ever available in practice. As ALS note in their introduction, a practical recourse is a MV optimizer that directly addresses estimation error for sign-constrained MV portfolio optimization.[5]

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[5] Michaud optimization (Michaud and Michaud 2008, Ch. 6) directly addresses estimation error in a sign-constrained MV efficient frontier framework.