A Better Way to Use Information

by

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European Pensions & Investment News
July 9, 2001

Abstract

In the second resampled efficiency article in *EPN*, Richard Michaud explains the importance of taking into account statistical errors in asset allocation decisions.

Resampled efficient optimization is a newly patented procedure that improves future investment performance, better forecasts risk and reduces the need to trade. In simple terms, resampled efficiency is a better way to use an investor's information. Resampled efficiency is based on research on how optimizers perform in rigorous statistical experiments. These "out-of-sample" tests prove that, on average, resampled efficiency improves performance.

Classical mean-variance asset allocation is unstable and ambiguous. Small changes in inputs or small differences in the risk levels of optimized portfolios may lead to very different asset allocations. Because the inputs are uncertain, the optimal asset allocation is ambiguous. Often, experienced investors find the resulting allocations unintuitive.

In practice, financial advisors constrain the optimization and manage the inputs to obtain allocations that seem reasonable. The end result may be useful for controlling risk and structuring the allocation so that it is consistent with investor objectives. However, investment value is often limited or even non-existent. Also, these constraints are often highly subjective and time consuming to implement.

Resampled efficiency solves these problems. Resampling methods provide a richer information set to limit instability and ambiguity. The resulting allocations are typically intuitive, stable, and have the added benefit of a much-reduced need to trade. Yet, because resampled efficiency is a generalization of classical methods, the benefits of current techniques are retained. Estimation error is the uncertainty associated with asset allocation estimates. Classical optimization uses inputs literally while resampled efficiency is sensitive to statistical uncertainty. The impact of estimation error on classical methods is far more serious than commonly understood in the investment community.

One way to understand estimation error is to consider a case where the two highest return assets in an asset allocation are equally risky with returns of 20% and 20.1%. Because classical optimization uses information literally, the maximum return optimal portfolio is 100% in the higher return asset. But this makes no investment sense. These assets are statistically equivalent and have essentially the same investment value. This demonstrates a fundamental flaw in how classical efficiency uses information.

The table provides an optimized portfolio illustration of the effect of estimation error on investment value for six asset classes for 10 years ending December 1999 and is an important illustration of its likely impact on optimized asset allocations.

Note that the classical portfolio has no allocation to small cap stocks. This is because the estimates are used literally and small caps are inferior to large caps. But there is likely to be little statistical difference between these assets and an investor may be well advised to include some small caps in their asset allocation. In contrast, resampled efficiency, includes a prominent component of small cap stocks. This is because resampled efficiency is sensitive to the level of estimation error and uses input data in a statistically robust manner. Note also that the resampled portfolio is more diversified and intuitively less risky.

How Resampled Efficiency Works

Asset allocation estimates of risk and return represent a forecast of the future. Resampling is a Monte Carlo based method for computing many alternative realizations of forecast returns.

The resampling process produces simulated returns and optimization inputs that are statistically consistent with the original estimates. These new input estimates are used to compute simulated efficient frontiers. Because of optimization instability, each simulated efficient frontier may be very different from the original frontier. These differences provide an understanding of the risks of optimizing and leads to a better way of computing optimal efficient portfolios.

Resampled efficient frontiers are an average of the simulated efficient frontier portfolios. These new portfolios are efficient relative to the many realizations of forecast return. Because resampled efficient portfolios are defined relative to many alternative ways the future may evolve, the allocations are less extreme, more intuitive, and risk estimation more reliable. Also, they are much more stable; small changes in the inputs typically lead to small changes in the resampled efficient portfolios, reducing the need to trade from one period to another.

One attractive characteristic of resampled efficiency is the ability to identify the significance and normal variation for each asset allocation. This is because each resampled efficient frontier provides an estimate of the optimal allocation for each asset. Consider the optimal large cap allocation in the table. The 10^{th} and 90^{th} percentile values are 31% and 67%. From this data we can say that the allocation to large caps is essential and that 31 to 67% is a normal range of variation.

One of the most reliable results taught by resampled efficiency is that many trades are unnecessary. This is because reasonable asset allocations are often statistically indistinguishable from optimal ones and don't require rebalancing. Resampled efficiency provides the ability to distinguish whether and when trading may be desirable.

Resampled efficient allocations are typically more intuitive, may not require ad hoc constraints, substantially reduce the need to trade and enhance risk estimation reliability. Most importantly, statistical tests show that on average, it is provably effective at improving investment value.

CLASSICAL VS. RESAMPLED OPTIMAL WEIGHTS[1]

Assets	Exp. Retn (%)	Std. Dev. (%)	Classical (%)	Resampled (%)
T-bills	4.8	0.4	0	6
Intrm Govt	7.1	4.3	36	18
LT Corp	8.3	6.4	0	12
LC Stock	17.7	13.4	64	53
SC Stock	15.7	17.4	0	10
Intl Stock	8.3	17.1	0	1

[1] The first two columns in the table display the annualized means and standard deviations of monthly data for six asset classes for 10 years ending December 1999. The last two columns display the classical and resampled optimal weights with equal annualized standard deviations of 9% in the middle of their respective frontiers.

The figures 31% and 67% do not appear in the table. They represent the range of asset allocation weights for the resampled efficient frontier allocation to large cap stocks of 53% or a "normal" range of values, where "normal" is defined as an 80% probability (90-10) for the resampled efficient frontier allocation.