



Richard Roll and Tests of the Capital Asset Pricing Model¹

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¹ Originally published in: *Heard on the Random Walk*, Bache, NYC, October 1979.

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Heard On The Random Walk

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The foundations of financial theory and modern investment management have been severely shaken with the publication of two papers by Richard Roll. In "A Critique of the Asset Pricing Theory's Tests," *Journal of Financial Economics*, 1977, and in "Ambiguity When Performance is Measured Using the Securities Market Line," *Journal of Finance*, 1978, Roll reaches the following conclusions: 1) no correct test of the Sharpe-Lintner (S-L) or Black versions of the Capital Asset Pricing Model (CAPM) has appeared and there is little possibility of a correct test in the future; 2) alpha cannot be used to measure inferior or superior investment performance, and beta is not a well defined measure of risk. If true, Roll's assertions imply that CAPM, the dominant paradigm of modern finance, has no empirical validation. His analysis also leads to a broad indictment of numerous investment management tools, including many large scale computer models, in widespread use in the investment community. This issue of Quantitative Investment Strategies (QIS) will focus on the problem of the empirical validation of CAPM as discussed in Roll (1977). We will leave to a future issue of QIS the critique of investment management techniques given in Roll (1978).

The basic result underlying much of Roll's analysis is the following fundamental efficient set

principle: There is a mathematical equivalence between mean-variance efficiency and return-beta linearity, i.e., knowledge of either mean-variance efficiency or return-beta linearity with respect to a given index portfolio implies the other. Part of the power of this result is due to the fact that it is essentially free of assumptions. It is therefore equally applicable to considerations involving prospective returns and the measurement of investment performance.

In many tests of CAPM, two versions of the model are examined. The original S-L derivation assumed the existence of a riskless borrowing and lending rate. In the Black version of CAPM, no such rate is assumed. In operational terms, both versions of CAPM imply a linear relationship between beta and return, but the S-L version of CAPM also implies a specific value for the intercept. It is the Black version of CAPM that has been generally considered supported by empirical data.

Many empirical tests of asset pricing theory have focused on determining the existence of a statistically significant linear relationship between beta and return. In the light of the efficient set principle, such attempts at testing CAPM are directly interpretable as tests of the mean-variance efficiency of the index portfolio.

However, the index portfolio is meant to proxy for the "market" portfolio, which CAPM hypothesizes is mean-variance efficient. Therefore, tests of CAPM which examine return-beta linearity are tests of the hypothesized mean-variance efficiency of the "market" portfolio.

The problem of the identifiability of the "market" becomes a critical consideration in interpreting the meaning of CAPM tests. This is true because in any sample of (ex ante or ex post) returns there will always be mean-variance efficient portfolios. By itself, the mean-variance efficiency or inefficiency of an index portfolio, says nothing of the mean-variance efficiency of the market portfolio. The validation of CAPM then turns on whether the index portfolio is a valid proxy for the "market." In short, return-beta linearity has empirical content as a test of CAPM only in the context of a well defined "market" portfolio. Any two observers who don't agree on the definition of the "market" can't be made to agree on the interpretation of empirical tests of CAPM.

Roll examines the empirical results of the three most famous tests of CAPM: Black, Jensen and Scholes (BJS), 1972; Fama and MacBeth (FM),

1973; Blume and Friend (BF), 1973. The last two tests use essentially similar approaches and will be discussed first. Both FM and BF test for the linearity of the return-beta relationship and for the value of the intercept. In general, test results are consistent with return-beta linearity and inconsistent with the S-L prediction for the intercept. In the context of Roll's analysis however, these empirical tests neither confirm nor deny the Black version of CAPM, since they don't address the issue of the validity of the index portfolio as a proxy for the "market." Also, further analysis reveals that the data can't be used to reject the S-L CAPM, since misspecification of the "market" portfolio in the regression will lead to misestimation of the intercept.

The BJS study is, in general, the foundation of the professional consensus that empirical data is consistent with the Black CAPM. One important reason is that their study contributed another piece of empirical data. The outcome of the study showed not only a return-beta linearity consistent with CAPM and an intercept value different from that predicted by the S-L CAPM, but also intertemporal variation in the intercept term. Whereas market portfolio misspecification may rationalize the misestimation of the intercept term, variability

of the intercept is inconsistent with S-L. As a result, the BJS study appears to separate the implications of the two competing versions of CAPM and confirm the Black model. Roll's analysis of the BJS testing procedure, however, revealed the following: Assume the S-L model is correct and that BJS did not choose the S-L market portfolio as the index portfolio. Assume also that the BJS index portfolio is mean-variance efficient. The resulting misspecification will lead to an intercept term that varies with the return of the S-L market portfolio. As a result, the BJS study cannot be used to accept or reject either version of the CAPM. In fact, Roll uses the BJS data to show that, under his assumed conditions, there was mean-variance efficient portfolio with a correlation of 0.89 with the BJS index portfolio which is exactly consistent with return-beta linearity and the S-L model. What is not known, however, is whether the S-L mean-variance efficient portfolio has the characteristics of a "market" portfolio, e.g., positive allocations for all assets.

In summary, Roll's application of the efficient set principle to the analysis of empirical tests of CAPM leads to the profoundly disturbing conclusion that there are no correct tests, and that the problem of the identifiability of the

"market" portfolio may imply that no valid tests are possible. Further, there is no justification for considering that empirical data are more consistent with either the Sharpe-Lintner or Black version of CAPM. Although serious criticism of CAPM has occurred before (e.g., Hakansson, *Journal of Financial and Quantitative Analysis*, 1971), none has appeared with the sweep and pervasiveness of Roll's critique. Upon reflection, the likely impossibility of the empirical validation of CAPM leads to the conclusion that CAPM may be operationally meaningless as a scientific hypothesis. However, empirical tests which attempt to describe cross-sectional differences in returns over time may have meaning apart from any particular scientific hypothesis. It is therefore likely that if the empirical data accumulated over the last decade has any theoretical significance, it will be in the context of a different financial model.

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October 22, 1979