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Out-of-Sample Tests of Resampled Efficiency

by

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Abstract

In first of two articles, Richard Michaud argues that a new form of asset allocation-resampled efficiency, improves the average reward-to-risk ratio of classical portfolios.

Resampled efficiency is, on average, provably effective at improving the investment value of optimized portfolios in practice. The purpose of this note is to review how claims of improved performance are often made in finance and why this claim, which is based on principles of modern statistics, is in a different category of tests and is far more reliable.

Investment managers typically use a back test framework to validate an investment strategy. This consists of choosing a relevant historical time period, an investable set of assets, beginning of period information, and subsequent returns. The strategy is applied in each period and subsequent realized returns averaged over the entire time period. The results show the performance of the strategy relative to a benchmark or competing strategy and are used to validate its benefits. Unfortunately, the results are not a proof of superiority and any conclusion is unreliable and may even be misleading.

The presence of an investor using the strategy in a market is likely to have an effect on market returns that are not captured by the test. Even if measures of trading costs or liquidity are included, they are usually unreliable. Most importantly, any back test is just one realization of the strategy's performance. To illustrate, consider tossing a fair coin ten times. On average, five heads are likely. However, any particular set of tosses may be unrepresentative. In the same way, any back test is just one way in which the strategy may have performed and the results are not reliable predictions of future performance.

Some back tests include holdout periods to test a strategy. The strategy is usually refined over some historical period and tested over a different or holdout period. Such tests are still unreliable because the data in the holdout period may be unrepresentative. In addition, if the strategy is chosen from a group of strategies based on superior performance in the holdout period, then the results violate the spirit of holdout tests and the back test is both unreliable and misleading.

Monte Carlo methods provide a rigorous framework for testing performance. A strategy can be tested and retested holding underlying assumptions constant in order to measure average performance. In a fundamental sense, the Monte Carlo framework turns back the clock again and again in order to compute average performance. This is the framework we use to compare the performance of classical vs. resampled efficiency.

Assume a set of assets with optimization inputs and an associated efficient frontier. The input data are called the true but unknown data set. The tests of classical and resampled efficiency performance are based on this data set.

Monte Carlo methods produce simulated returns that are statistically consistent with the original input data set. By simulating many returns, we can compute a new set of statistically consistent optimization inputs and classical and resampled efficient frontiers. Now evaluate the performance of portfolios on the simulated efficient frontiers in terms of the original data set. By doing this we are measuring how the optimized portfolios actually performed in the context of the true but unknown data, the data that would actually determine performance over the period. Repeat this procedure many times.

Averaging the risk and return of the optimized portfolios in terms of the original data provides the average performance of classical and resampled optimized portfolios.

Monte Carlo tests confirm that resampled efficiency, on average, improves the performance of classical optimized portfolios. The basic result is that, on average, classical efficient portfolios tend to have approximately the same return, but significantly more risk, than their associated resampled efficient portfolios. There are two ways to use these results. One way is to show that resampled efficiency improves the average reward-to-risk ratio of associated classical efficient portfolios. In other words, you are better off using the resampled version of the classical efficient frontier portfolio that you were going to use. A second way is to show that resampled efficient portfolios have a higher average reward-to-risk ratio for the same risk of classical efficient portfolios.

Resampled efficiency improves, on average, portfolio performance relative to classical optimized portfolios. In contrast to back tests of investment strategies, these Monte Carlo tests are rigorous and reliably measure performance differences in practice. While no one can guarantee that any particular resampled efficient portfolio will perform better than a classically optimized one, on average, the resampled efficient investor will do better. The tests confirm that resampled efficiency is a better way of using an investor's information for producing optimized equity portfolios all other things the same.

While performance improvement is a significant issue, it is not the only benefit of resampled efficiency relative to classical methods. Resampled efficiency improves investment performance because it is a more stable decision making framework, reduces trading without benefit, and more reliably estimates risk. These and other considerations make resampled efficiency the method of choice for portfolio optimization in practice.

In the next issue of *EPN*, Richard Michaud details the following key benefits to resampled efficiency:

- Resampled efficiency is a better way to use an investor's information
- Classical mean-variance asset allocation unstable and ambiguous
- Resampling methods provide a richer information set to limit instability and ambiguity
- The resampling process produces simulated returns and optimization inputs that are statistically consistent with the original estimates